Period 3, Feb 19, 2025

$$\int \frac{\csc x \cot x}{\sqrt[4]{\csc x}}\,dx =$$

$u = \csc x \qquad du = -\csc x \cot x\,dx$

$$\frac{du}{-\csc x \cot x} = dx$$

$$\int \frac{\cancel{\csc x}\,\cancel{\cot x}}{u^{1/4}} \cdot \frac{du}{-\cancel{\csc x}\,\cancel{\cot x}} = \int -u^{-\frac{1}{4}}\,du = -\int u^{-\frac{1}{4}}\,du = -\cdot \frac{4}{3} \cdot u^{-\frac{1}{4}+1=\frac{3}{4}} + C$$

$$\csc x^{3/4} = \csc\left(x^{\frac{3}{4}}\right) \neq (\csc x)^{3/4} = \csc^{\frac{3}{4}} x$$

$$-\frac{4u^{3/4}}{3} + C = \frac{-4(\csc x)^{3/4}}{3} + C$$

$$= \frac{-4\csc^{\frac{3}{4}} x}{3} + C$$

$$\int \frac{\sec x \tan x}{\sqrt[4]{\sec x}}\,dx =$$

$u = \sec x$

$$du = \sec x \tan x\,dx \Rightarrow \frac{du}{\sec x \tan x} = dx$$

$$\int \frac{\cancel{\sec x}\,\cancel{\tan x}}{\sqrt[4]{u}} \cdot \frac{du}{\cancel{\sec x}\,\cancel{\tan x}} = \int \frac{1}{u^{1/4}}\,du = \int u^{-\frac{1}{4}}\,du = \frac{4}{3} \cdot u^{-\frac{1}{4}+1=\frac{3}{4}} + C$$

$$\frac{4}{3}(\sec x)^{\frac{3}{4}} + C$$

$$\frac{4\sec^{\frac{3}{4}} x}{3} + C = \frac{4\sqrt[4]{\sec^3 x}}{3} + C$$

$$\int \frac{2x}{1+x^4}\,dx$$

$$\boxed{\begin{array}{l} u = \cancel{1+}x^4 \\ du = 4x^3\,dx \\ \dfrac{du}{4x^3} = dx \qquad \int \dfrac{2x}{u} \cdot \dfrac{du}{4x^{3\,2}} \end{array}}$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$u = x^2$

$u^2 = x^4$

$du = 2x\,dx$

$$\frac{du}{2x} = dx$$

$$\int \frac{\cancel{2x}}{1+u^2} \cdot \frac{du}{\cancel{2x}} = \int \frac{du}{1^2+u^2} = \frac{1}{1}\arctan\frac{u}{1} + C = 1\arctan x^2 + C$$

$$\int \frac{-2x}{1+x^4}\,dx$$

$u = x^2$

$u^2 = x^4$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$\int \frac{\cancel{-2x}}{1+u^2} \cdot \frac{du}{\cancel{2x}} = -\int \frac{du}{1+u^2}$$

$-\arctan x^2 + C$

$$\int \frac{e^x}{6e^x - 5}\,dx \Rightarrow \int \frac{\cancel{e^x}}{u} \cdot \frac{du}{6\cancel{e^x}} = \int \frac{du}{6u} = \frac{1}{6}\int \frac{du}{u} = \frac{1}{6}\ln|u| + C$$

$u = 6e^x - 5$

$du = 6 \cdot e^x\,dx - 0$

$\frac{du}{6e^x} = dx$

$$\boxed{\int \frac{du}{u} = \int u^{-1}\,du = \ln|u| + C}$$

$\frac{1}{6}\ln|6e^x - 5| + C$

$\ln\left|\sqrt[6]{6e^x - 5}\right| + C$

$$\int \frac{e^x}{4 - 3e^x}\,dx$$

$u = 4 - 3e^x$

$du = 0 - 3e^x \cdot dx$

$\frac{du}{-3e^x} = dx$

$$\int \frac{\cancel{e^x}}{u} \cdot \frac{du}{-3\cancel{e^x}}$$

$$-\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|4 - 3e^x| + C = \ln\left|\frac{1}{\sqrt[3]{4 - 3e^x}}\right| + C$$

$$\int 5x^3(4-7x^4)^{12}\,dx$$

$u = 4-7x^4$

$du = 0 - 7\cdot 4\,x^{4-1}\,dx = -28x^3\,dx$

$\dfrac{du}{-28x^3} = dx$

$$\int 5\cancel{x^3}\cdot u^{12}\cdot\frac{du}{-28\cancel{x^3}} = \frac{-5}{28}\int u^{12}\,du = \frac{-5}{28}\cdot\frac{1}{13}\cdot u^{12+1=13} + C$$

$$\frac{-5}{28\cdot 13}(4-7x^4)^{13} + C = \frac{-5}{364}(4-7x^4)^{13} + C$$

$$\int 7x^3(9-4x^4)^{11}\,dx$$

$u = 9-4x^4$

$du = 0 - 4\cdot 4\cdot x^{4-1}\,dx = -16x^3\,dx$

$\dfrac{du}{-16x^3} = dx$

$$\int 7\cancel{x^3}\cdot u^{11}\cdot\frac{du}{-16\cancel{x^3}} = \int\frac{7}{-16}u^{11}\,du = -\frac{7}{16}\cdot\frac{1}{12}\cdot u^{11+1=12} + C$$

$$\frac{-7}{192}(9-4x^4)^{12} + C$$

#2, #1, 21, 17, 7, 18, 8, 16, 11, 10

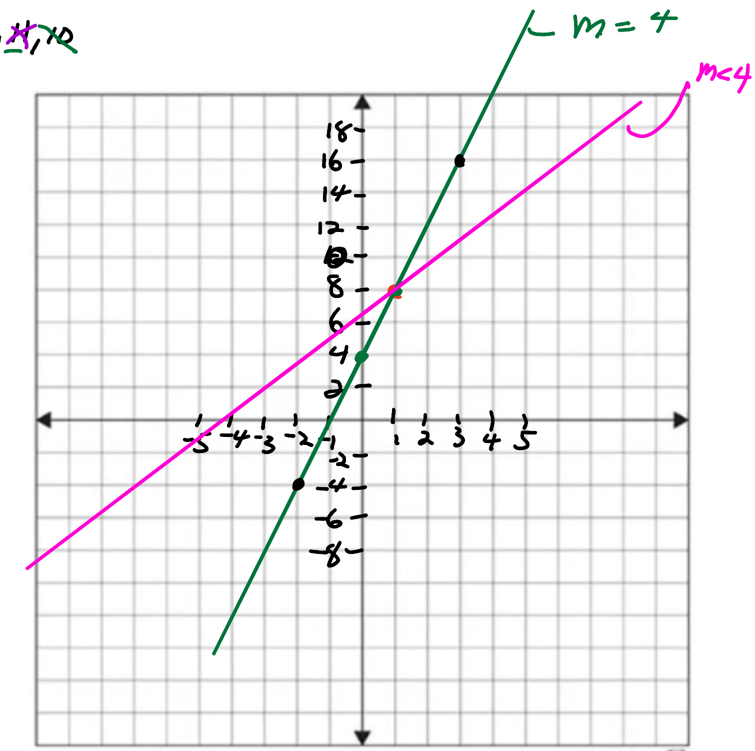
$F(1)=8$ $F'(x)\le 4$

$y-y_1=m(x-x_1)$

$y-8=4(x-1)$

$y-8=4x-4$
$+8 \qquad +8$

$y=4x+4$



$F(3)\le 16$

$F(-2)\ge -4$

$F(0)\ge 4$

MVT

② $g(x)=\frac{1}{4}x^4+\frac{2}{3}x^3-\frac{3}{2}x^2+4$

③ C.# $g'(x)=0$ or ϕ $\quad g'(x)=x^3+2x^2-3x=x(x^2+2x-3)$

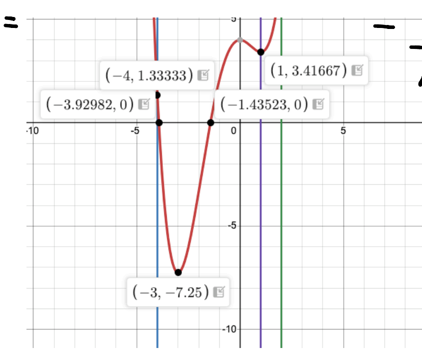
$0=x(x+3)(x-1)$

critical # $x=0, x=-3, x=1$

$g(0)=\frac{1}{4}(0)^4+\frac{2}{3}(0)^3-\frac{3}{2}(0)^2+4=4$

$g(-3)=\frac{1}{4}(3)^4+\frac{2}{3}(3)^3-\frac{3}{2}(-3)^2+4=\frac{81}{4}+\frac{54}{3}-\frac{27}{2}+4=\frac{243}{12}-\frac{216}{12}-\frac{162}{12}+\frac{48}{12}-\frac{87}{12}=-7.25$

$g(1)=\frac{1}{4}+\frac{2}{3}-\frac{3}{2}+4=\frac{3}{12}+\frac{8}{12}-\frac{18}{12}+\frac{48}{12}=\frac{41}{12}=$



$g(-1)=\frac{25}{12}$

$g(2)=7\frac{1}{3}$

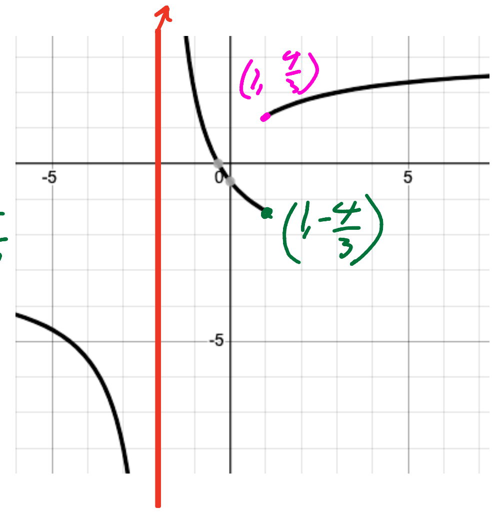
$g(-4)=\frac{1}{4}(-4)^4+\frac{2}{3}(-4)^3-\frac{3}{2}(-4)^2+4=1\frac{1}{3}$

$g(1)=\frac{41}{12}$

⑦

$$h(x)=\frac{|x-1|(3x+1)}{(x-1)(x+2)}=$$

ⓐ V. A. $x=-2$ $x=1$ discontinuous



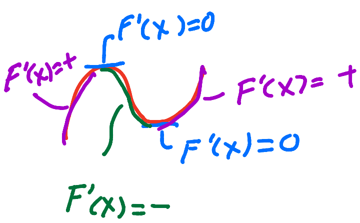
$$\lim_{x\to1^+} h(x)=\frac{|\cancel{1.01}-1|\,(3(1)+1)}{(\cancel{1.01-1})(1+2)}=\frac{1\cdot4}{3}=\frac{4}{3}$$

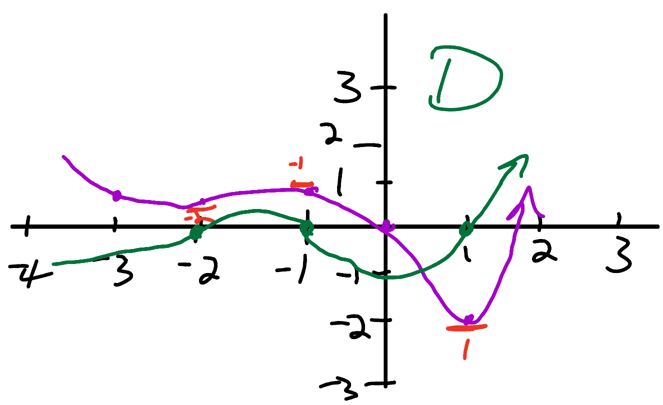
$$\lim_{x\to1^-} h(x)=\frac{\boxed{\frac{|.99-1|}{(.99-1)}}\,(3(1)+1)}{(1+2)}=-1\cdot\frac{4}{3}=\frac{-4}{3}$$

-1

#10

F(x) with F'(x)   F'(x)





⑪ $F(x)=-2x^2+16x-3$ MVT $[-2,3]$

$F(-2)=-2(-2)^2+16(-2)-3=-8-32-3=-43$

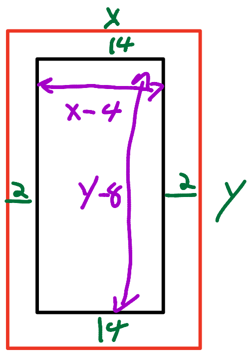
$F(3)=-2(3)^2+16(3)-3=-18+48-3=27$

$(-2,-43)$ $(3,27)$ $\frac{-43-27}{-2-3}=\frac{-70}{-5}=14$

$F'(x)=-4x+16=14 \Rightarrow -4x=-2$

$x=\frac{1}{2}$

13.



$$x \cdot y = 50 \Rightarrow y = \frac{50}{x}$$

$$\text{Printed Area} = (x-4)(y-8)$$

$$= (x-4)\left(\frac{50}{x}-8\right)$$

Printed Area $= (1, 2)$

$$\text{Printed Area} = 50 - 8x - \frac{200}{x} + 32$$

$$PA = 82 - 8x - 200x^{-1}$$

$$\frac{dPA}{dx} = 0 - 8 + 200x^{-2}$$

$$0 = -8 + \frac{200}{x^2}$$

$$x^2 \cdot 8 = \cancel{\frac{200}{x^2} \cdot x^2}$$

$$8x^2 = 200$$

$$x^2 = \frac{200}{8} = 25$$

$$x = 5$$

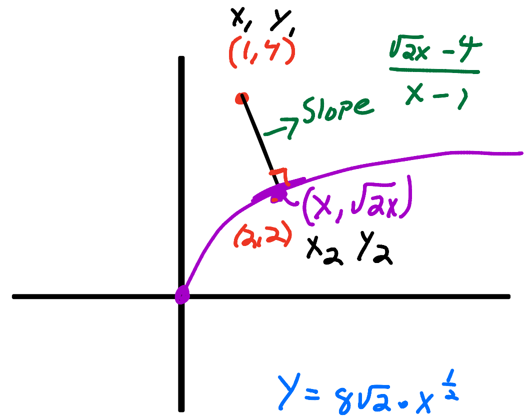
$$y = \frac{50}{x} = \frac{50}{5} = 10$$

$x = 5$

$y = 10$

(18) $y=\sqrt{2x}$ $\quad$ $x=2$ $\quad$ $\sqrt{2\cdot 2}=y$

$(1,4)$ $\quad$ $y=2$



$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

For max min $\frac{dd}{dx}=0$ or $\varnothing$

$$d^2=\left(\sqrt{(x-1)^2+(\sqrt{2x}-4)^2}\right)^2$$

$$d^2=(x-1)^2+(\sqrt{2x}-4)^2=x^2-\cancel{2x}+1+\cancel{2x}-8\sqrt{2x}+16$$

$y=8\sqrt{2}\cdot x^{\frac{1}{2}}$

$\frac{dy}{dx}=8\cdot\sqrt{2}\cdot\frac{1}{2}x^{-\frac{1}{2}}$

$\frac{8\sqrt{2}}{2\sqrt{x}}$

$$d^2=x^2-8\sqrt{2}\cdot x^{\frac{1}{2}}+17$$

$$2d\frac{dd}{dx}=2x-8\sqrt{2}\cdot\frac{1}{2}x^{-\frac{1}{2}}+0$$

$$2d\cdot 0=2x-\frac{4\sqrt{2}}{\sqrt{x}}$$

$$0=2x-\frac{4\sqrt{2}}{\sqrt{x}}\Rightarrow \frac{\cancel{\sqrt{x}}}{2}\cdot\frac{4\sqrt{2}}{\cancel{\sqrt{x}}}=\frac{\cancel{2}x\sqrt{x}}{\cancel{2}}$$

$$2\sqrt{2}=x\sqrt{x}$$
$$2^{3/2}=x^{3/2}$$
$$x=2$$

Find using Slope

$y=\sqrt{2}\cdot x^{\frac{1}{2}}$

$\frac{dy}{dx}=\sqrt{2}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x}}=\frac{\sqrt{2}}{2\sqrt{x}}$

$\frac{dy}{dx}=\frac{1}{\sqrt{2x}}\cdot\cancel{\frac{2}{2}}$

Slope of Line

$\frac{\sqrt{2x}-4}{x-1}$

Slope of curve

$\frac{\sqrt{2}}{2x}$

$1(\sqrt{2x}-4)=-\sqrt{2x}(x-1)$

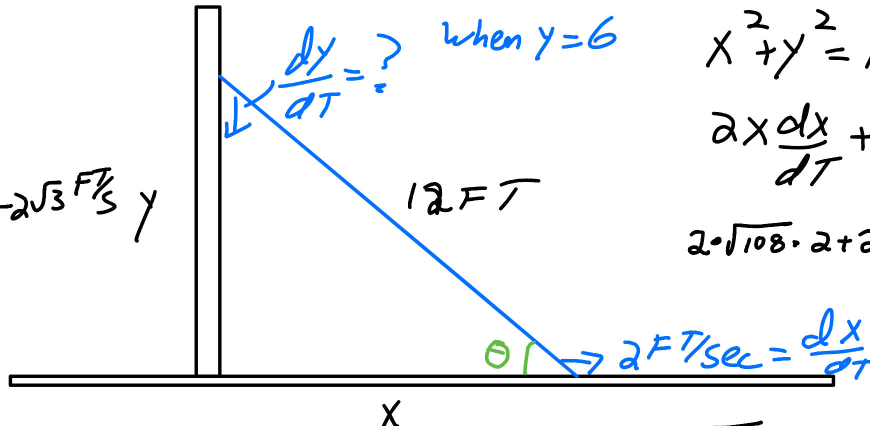
$\cancel{\sqrt{2x}}-4=-x\sqrt{2x}+\cancel{\sqrt{2x}}\Rightarrow\frac{-4}{\sqrt{2}}=\frac{-x^{\frac{3}{2}}\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}\Rightarrow -2\sqrt{2}=-x^{\frac{3}{2}}\Rightarrow -2^{\frac{3}{2}}=x^{\frac{3}{2}}$

$\frac{\sqrt{2x}-4}{x-1}=\frac{-\sqrt{2x}}{1}$

$x=2$

21.

$\frac{dy}{dT} = -2\sqrt{3}\ FT/s$



when $y=6$

$x^2 + y^2 = 12^2$

$2x\frac{dx}{dT} + 2y\frac{dy}{dT} = 0$

$2 \cdot \sqrt{108} \cdot 2 + 2 \cdot 6 \cdot \frac{dy}{dT} = 0$

$4\sqrt{108} + 12\frac{dy}{dT} = 0$

$x^2 + 6^2 = 144$

$x^2 = 108$

$x = \sqrt{108} = 6\sqrt{3}$

$12\frac{dy}{dT} = -4 \cdot 6\sqrt{3}$

$\frac{dy}{dT} = \frac{-24\sqrt{3}}{12} = -2\sqrt{3}$

Find $\frac{d\theta}{dT}$ when $y=6$, $x = 6\sqrt{3}$, $\frac{dx}{dT} = 2\ FT/sec$

$\frac{dy}{dT} = -2\sqrt{3}\ FT/sec$

You can use

$\tan\theta = \frac{y}{x} \Rightarrow \sec^2\theta\frac{d\theta}{dT} = \frac{\frac{dy}{dT}\cdot x - y\cdot\frac{dx}{dT}}{x^2} \Rightarrow \left(\frac{12}{\sqrt{108}}\right)^2\frac{d\theta}{dT} = \frac{-2\sqrt{3}\cdot 6\sqrt{3} - 6\cdot 2}{(6\sqrt{3})^2}$

$\cos\theta = \frac{x}{12} \Rightarrow -\sin\theta\frac{d\theta}{dT} = \frac{1}{12}\frac{dx}{dT} \Rightarrow \frac{-6}{12}\frac{d\theta}{dT} = \frac{1}{12}\cdot 6\sqrt{3}$ Find $\frac{d\theta}{dT}$

$\sin\theta = \frac{y}{12}$

$\cos\theta\frac{d\theta}{dT} = \frac{1}{12}\frac{dy}{dT} \Rightarrow \cancel{\frac{12}{x}}\cancel{\frac{x}{12}}\frac{d\theta}{dT} = \frac{1}{12}\cdot -2\sqrt{3}\cdot\frac{12}{x}$

$\frac{d\theta}{dT} = \frac{1}{12}\cdot -2\cancel{\sqrt{3}}\cdot\frac{12}{6\cancel{\sqrt{3}}} = \frac{-2\cdot\cancel{12}}{\cancel{12}\cdot 6} = -\frac{1}{3}\ R/s$

$\frac{d\theta}{dT} = -\frac{1}{3}\ R/s$